Multistage Production under Trade Policy Uncertainty

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Abstract

The recent increase in trade policy uncertainty affects a variety of industries. In particular, uncertainty is important for industries whose final good is produced in multiple stages that are located across different countries. These industries are the most concerned about trade policy and reduce investment during uncertainty periods. This paper analyzes trade policy uncertainty in a two-country dynamic, stochastic, general equilibrium model with multistage production where a firm's decisions today depends on the future tariff path. Studies with one stage of production that measure the effect of trade policy find that uncertainty, as a second moment shock, does not play a big role in explaining the changes observed in the economy. Introducing multistage production, which generates a magnified response of trade to tariff changes, provides a better mechanism to analyze the role uncertainty in future tariffs plays in the economy.

1 Introduction

New trade deals, trade wars, and renegotiations of trade agreements have increasingly become the focus of investors, firms, politicians, and market participants. These events have resulted in a more uncertain outlook for global trade. Before this surge of trade developments, there had been limited volatility in trade policy, and thus limited study of the impact of uncertainty regarding trade agreements on the economy[1].

A recent study by Caldara, Iacovello, Molligo, Prestipino, and Raffo (2019) finds empirical evidence of an increase in trade policy uncertainty for the period 2015-2018[2]. They document that in the beginning on 2015, less than 3% of the firms discussed trade policy uncertainty, and by the end of 2018 the number had increased to 15%[3].

Additionally, they find a strong sectoral variation in the increase of trade policy uncertainty. As seen in figure 2, as trade policy uncertainty increases there are some industries more concerned about it than others. Industries that record the highest level of concern are nondurables, chemicals, manufacturing, and durables. Also, as figure 3 shows, they find that these types of industries register low investment during the rise of uncertainty. These industries, which include motor vehicles, chemicals, and textiles, have a higher degree of vertical specialization[4] for the US according to Hummels, Ishii, and Yi (2001). A high degree of vertical specialization means that the final good is produced in multiple stages located across different countries.

To understand the effect trade policy uncertainty has in an economy, this paper analyzes a two-country dynamic, stochastic, general equilibrium model with multistage production where the firm's decisions today depends on the future tariff path. It follows Yi (2010) static, two-staged production model, and introduces a sunk cost[5] for stage two firms to import stage one inputs for its production, creating a scope for trade policy uncertainty to affect macroeconomic dynamics and trade flows. Following Steinberg B. (2019), this paper

[1] There are important exceptions, with recent papers focusing on trade uncertainty. For instance, Handley and Limao (2017) estimate and quantify the impact of trade policy on China's export boom to the United States following its 2001 WTO accession, and Steinberg B. (2019) study the effects of trade uncertainty associated with the Brexit referendum.

[2] They empirically measure trade policy uncertainty and its effects by creating a firm level measure of trade policy uncertainty based on text analysis of earnings calls of publicly listed companies where they look for terms related to trade uncertainty.

[3] See evidence in figure 1

[4] They define that vertical specialization occurs when: (1) a good is produced in two or more sequential stages; (2) two or more countries provide value-added during the production of the good; and (3) at least one country must use imported inputs in its stage of the production process.

[5] Sunk costs are modeled in a similar was as Alessandria and Choi (2007).

measures the impact of uncertainty by comparing an equilibrium with perfect foresight to a stochastic equilibrium with uncertainty in future tariffs. To isolate the effect uncertainty has in the economy, the perfect foresight equilibrium is evaluated along the expected value of the tariffs used in the stochastic equilibrium.

The main contribution of this paper is the introduction of multistage production to an environment with trade policy uncertainty. The emerging literature that analyzes trade policy uncertainty has only done it through the lens of a one stage production model. They have found that uncertainty, as a second moment shock, does not play a big role in explaining the changes observed in the economy associated to a change in trade policy.

Introducing uncertainty in future tariffs in the multistage model with sunk costs generates the option value of waiting, as studied by Dixit (1989). Given a low tariff shock, firms will delay the decision to start importing due to a possible future tariff increase. Similarly, given a high tariff shock, firms will delay the decision to stop importing, due to a possible tariff decrease. But waiting is costly for the economy.

So far, this paper presents a simplified environment with one period uncertainty. In this framework stage two firms in the stochastic equilibrium decide to wait until the uncertainty period is resolved to make investment decisions. Waiting is costly because in the perfect foresight equilibrium, firms anticipate the change in tariffs, which allows the households to smooth consumption.

Further study, using the full structure of the model and multi-period uncertainty, will show other mechanisms where trade policy uncertainty is costly for the economy. Depending on the number of periods uncertainty is present, stage two firms may chose to become importers, pay the associated sunk cost, and never be able to import. Or stage two firms may continue to import when they shouldn't. This scenarios create additional costs of uncertainty in the economy.

This paper combines two literatures on trade policy. The first studies the effect of trade policy uncertainty in the context of one stage production models. Among this studies is Caldara, Iacovello, Molligo, Prestipino, and Raffo (2019) who develop a two country New Keynesian dynamic, stochastic, general equilibrium one stage model to measure the impact trade policy uncertainty has in the economy. They find that higher expected value of the tariffs account for the largest part of the change in the economy, and the uncertainty effect is only binding in an economy with nominal rigidities. Steinberg B. (2019) studies the effects of trade uncertainty associated with the Brexit referendum under a dynamic, stochastic, general equilibrium one stage model with forward-looking decisions about export participation. Even though he documents a substantial welfare loss from Brexit, the uncertainty effect found is small[6].

The second literature studies the importance of multistage production in assessing the impact of tariffs. Important examples, are Yi (2010) and Johnson C. and Moxnes (2019). They show that multistage production allows for a framework where trade costs change the underlying nature of production, which allows for firms to allocate its production chain across countries. It is able to explain the large, nonlinear response of trade flows to changes in trade costs that one stage production models fail to observe. Introducing multistage production to a dynamic model with trade policy uncertainty generates a better mechanism to understand the reach this type of uncertainty has in an economy.

In addition to the studies already described, this paper also contributes to the related literature on trade dynamics with heterogeneous firms and endogenous export participations incorporating features from the new exporter dynamics literature, which emphasizes sunk costs of starting to export (Alessandria and Avila (2017), Alessandria and Choi (2007), Alessandria and Choi (2019), Alessandria et al. (2018), Das et al. (2007), and Atkenson and Burstein (2010)). It is also related to an active literature on input sourcing and multi-stage production (Fally and Hillberry (2019), Antrás and De Gortari (2017), Gumpert et al.

[6]Steinberg B. (2019) finds Brexit's consumption-equivalent welfare losses are between 0.4 and 1.2 percent depending on how much post-Brexit trade costs rise. The welfare cost of uncertainty about Brexit is small, however, accounting for less than a quarter of a percent of the overall welfare cost.

(2018), among others.

The paper proceeds as follows. The model is described in section 2. Section 3 includes a description of the solution method. Preliminary results on the simplified environment are presented in section 4. Section 4 concludes.

2 Model

This section lays out the elements of the dynamic multistage model. It begins by describing the economic environment and then defines the model equilibrium. Then describes the methodology used to model uncertainty and finalizes with a discussion of the main elements of the model.

2.1 Economic Environment

To analyze the effect uncertainty has on an economy with multistage production, this section describes a two-country dynamic, stochastic, general equilibrium model with two stages of production. Each country, $i = \{1, 2\}$, is populated by a representative household and a unit measure of heterogenous firms for each stage of production. Each variety z along the unit interval $[0, 1]$ has a stage one and a stage two good. This goods are tradable, and both countries possess the technology for producing all varieties in both stages.

Stage one variety z is produced by a stage one firm using labor and a composite good. The variety z of stage two good is produced using that same variety z of stage one good and labor by a stage two firm. The unit measure of stage two varieties, $z \in [0, 1]$ is aggregated to produce the composite good used as input in stage one, and the final good consumed by the representative households.

For each variety $z \in [0, 1]$ there are two firms, one in each country, competing to produce the stage one variety, and two stage two firms competing for the final consumption market. Stage one goods are produced in each country by competitive firms. Stage two firms take into account the demand they face from the final household, and engage in Bertrand competition with their direct competitor from the other country. Additionally, stage two firms face the dynamic choice whether to import stage one good from another country or to source it locally. It's a dynamic choice because the decision to import requires the payment of an initial one time only cost, f_0, and a per-period fixed cost to maintain its status as an importer, f_1. At the beginning of the period stage two firms lose their status as importers at exogenous rate δ.

Stage two firms that want to import its input from stage one, and the households that want to import the stage two variety good, must pay a tariff, τ. In each period of time, the economy may experience a shock in tariffs, τ_t. The history of tariff shocks from period zero to period t will be denoted as $\tau^t = (\tau_0, \tau_1, ..., \tau_t)$. The probability of a history τ^t, conditional on the information available at period 0, is defined as $\pi(\tau^t|\tau_0)$.

The timing of the model within each period is as follows.

1. First the tariff, τ, and δ shock is revealed.
2. Then stage two firms decide their status as an importer. If they start importing stage one goods, they must pay the sunk cost.
3. Finally, production and consumption takes place.

This timing is relevant for the model due to the structure of the competition stage two firms face. More on this later.

2.1.1 Households

Representative households in each country, $i = \{1, 2\}$, demand the unit measure varieties of stage two goods, $x_2^i(z, \tau^t)$, and aggregate them to produce the final consumption good,

$C^i(\tau^t)$, and the nontraded aggregate composite good, $N^i(\tau^t)$, used as inputs in stage one production.

$$C^i(\tau^t) + N^i(\tau^t) = \left(\int_0^1 x_2^i(z,\tau^t)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{1-\epsilon}} \tag{1}$$

Thus the price index for the final and composite good, $P^i(\tau^t)$ is given by aggregating the final price of stage two good varieties country i faces, $p_2^i(\tau^t)$.

$$P^i(\tau^t) = \left(\int_0^1 p_2^i(\tau^t)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}} \tag{2}$$

The household has preferences over the final consumption good,

$$\sum_{t=0}^{\infty} \sum_{\tau^t} \beta^t \pi(\tau^t|\tau_0) u(C^i(\tau^t)) \tag{3}$$

subject to its budget constraint

$$\int_0^1 p_2^i(z,\tau^t) x_2^i(z,\tau^t) dz = w^i(\tau^t) L_t^i + P^i(\tau^t) N^i(\tau^t) + \Pi^i(\tau^t) + T^i(\tau^t) \tag{4}$$

where the household pays final price $p_2^i(z,\tau^t)$ for stage two good z, and receives resources from its labor supply, L_t^i, the production of the aggregate composite good, $P^i(\tau^t)N^i(\tau^t)$, which sells to stage one firms, the aggregate profits from all stage two firms in country i net of fixed and sunk costs, $\Pi^i(\tau^t)$, and the tariffs collected from the imports of stage one and two goods, $T^i(\tau^t)$.

2.1.2 Stage one production

Stage one goods in each country $i \in \{1,2\}$ are produced from labor and the composite good:

$$y_1^i(z,\tau^t) = A_1^i(z) l_1^i(z,\tau^t)^{1-\theta_1} N^i(z,\tau^t)^{\theta_1} \qquad z \in [0,1] \tag{5}$$

where $A_1^i(z)$ is countries i's productivity associated with stage one good z, and $l_1^i(z,\tau^t)$ and $N^i(z,\tau^t)$ are the demanded units of labor and aggregate composite good to produce $y_1^i(z,\tau^t)$.

Firms are competitive, thus maximizing profits every period, subject to their technology.

$$\max_{l_1^i(z,\tau^t),N^i(z,\tau^t)} q_1^i(z,\tau^t)y_1^i(z,\tau^t) - w^i(\tau^t)l_1^i(z,\tau^t) - P^i(\tau^t)N^i(z,\tau^t) \tag{6}$$

where $q_1^i(z,\tau^t)$ is the price stage one firms receive, $w^i(\tau^t)$ is the wage in country i, and $P^i(\tau^t)$ is the price of the composite good.

Because firms are perfectly competitive in this stage, the price is equal to its marginal cost

$$q_1^i(z,\tau^t) = \frac{w^i(\tau^t)^{1-\theta_1}P^i(\tau^t)^{\theta_1}}{(1-\theta_1)^{1-\theta_1}\theta_1^{\theta_1}A_1^i(z)} \tag{7}$$

2.1.3 Stage two production

The problem for the unit measure of stage two firms in each country can be decomposed into a dynamic and a static choice.

1. Stage two firms face a dynamic choice whether to import stage one goods from the other country, $m = 1$, and pay the associated sunk and fixed costs, or source the stage one good from its own country, $m = 0$.

2. Firms maximize their static profits subject to the dynamic choice, the technology, and the demand they face from the representative households.

Before stating the problem the firms solve, following is a discussion of the type of information needed for firms to solve the problem each period.

Productivity vector Each variety z has a vector $Z = \{A_1^1(z), A_1^2(z), A_2^1(z), A_2^2(z)\}$ of associated productivities. For a stage two firm, its overall productivity depends on its production productivity, $A_2(z)$, and the productivity the firm sources its stage one input from, $A_1(z)$. Whether the stage two firm produces or not depends on its relative productivity compared with its direct competitor's. For example, a highly productive stage two firm may not produce because its competitor happens to be more productive. Thus, a stage two firms needs to know not only its own productivity, but the productivities of all the potential firms it can source the stage one good from, and the productivity of its competitor.

Aggregate state. The dynamic choice stage two firms face depends on the difference in the expected profits it would obtain from importing or sourcing from its own country. If the difference in the discounted profits from importing is greater than the value of the sunk cost, they decide to import.

Profits today depend on the current tariff shock, and aggregate prices. To calculate today's aggregate prices, firms need to know which firms decide to import today, and have to pay the fixed cost, and which firms are new importers, and will also have to pay the sunk cost. To know what firms are new importers today, firms need to know which firms imported yesterday and survived the δ shock. This information is represented in the aggregate state, S, which is the distribution of firms that entered the period as importers.

1. Static choice. Stage two firms must maximize the per-period profits, $\Pi^i(Z, m; \Lambda)$. Profits depend on the (i) aggregate vector of productivities $Z = \{A_2^1(z), A_2^2(z), A_1^1(z), A_1^2(z)\}$ that includes stage two productivity for variety z in each country, and stage one productivity for the same variety in each country, (ii) today's dynamic choice whether to import, $m = 1$, or not, $m = 0$, and (iii) the aggregate state of the economy, $\Lambda = (\tau, S)$. Λ is composed of the tariff shock today, τ, and S, the distribution fo firms that entered the period as importers. Aggregate prices and quantities depend on the aggregate state of the economy, S.

Given this information, the firm in country i choses the price for the stage good sold in country 1 and 2, $p_2^{i1}(Z; \Lambda), p_2^{i2}(Z; \Lambda)$, and its inputs for production: stage one good z and labor, $x_1^i(Z;\Lambda), l_2^i(Z;\Lambda)$.

$$\begin{aligned} \Pi^i(Z,m;\Lambda) = \max_{p_2(Z;\Lambda),x_1^i(Z;\Lambda),l_2^i(Z;\Lambda)} \quad & p_2^{i1}(Z)y_2^{i1}(Z) + p_2^{i2}(Z)y_2^{i2}(Z) \\ & - x_1^i(Z)\Big(mp_1^i(Z;\Lambda) + (1-m)q_1^i(Z;\Lambda)\Big) \\ & - w^i(\Lambda)l_2^i(Z) \end{aligned} \tag{8}$$

where $y_2^{i1}(Z), y_2^{i1}(Z)$ represent the production allocated for country 1 and 2 respectively for stage two good z. If the firm decides to become an importer, $m = 1$, it will have access to the economy's lowest cost producer of stage one good z, given by $p_1^i(Z;\Lambda)$. Otherwise, it will have to source stage one good z from its own country, with price $q_1^i(Z;\Lambda)$.

Firms maximize their profits subject to technology, and the demand they face by the representative household in each country.

$$y_2^{i1}(Z) + y_2^{i2}(Z) = A_2^i(z)l_2^i(Z)^{1-\theta_2}x_1^i(Z)^{\theta 2} \tag{9}$$

$$p_2^i(Z;\Lambda) = P^i(\Lambda)\left(\frac{(C^i(\Lambda) + N^i(\Lambda))}{x_2^i(Z;\Lambda)}\right)^{1/\epsilon} \qquad i \in \{1,2\} \tag{10}$$

where $x_2^1(Z;\Lambda), x_2^2(Z;\Lambda)$ are the demands for stage two good z in country 1 and 2. The demands depend on the aggregate level of the final good, $C^i(\Lambda)$, the aggregate composite good, $N^i(\Lambda)$, the price index for the final and composite good, $P^i(\Lambda)$, and the final price of stage two good z for household from country i, $p_2^i(Z;\Lambda)$.

2. Dynamic Choice. At the beginning of each period, every existing importer has a probability δ of losing its status as an importer exogenously, and with probability $(1-\delta)$ the firm maintains its status as an importer.

Firm's enter the period with an import status given by m_{-}, that reflects the firm's decision to import yesterday and the probability that the firms survived today's shock, δ. Given the firm's import status, m_{-}, and today's dynamic choice m, firms will pay the sunk cost, f_0, and the per-period fixed cost, f_1. The dynamic problem of the firm is the following.

$$\begin{aligned} V^i(Z, m_{-}; \Lambda) = \max_{m \in \{0,1\}} \; & \Pi^i(Z, m; \Lambda) - w^i(\Lambda)\Big(m(1-m_{-})f_0 + mf_1\Big) \\ & + \delta \sum_{\tau'} Q^i(\tau'|\tau)V^i(Z, 0; \Lambda') + (1-\delta)\sum_{\tau'} Q^i(\tau'|\tau)V^i(Z, m; \Lambda') \end{aligned} \tag{11}$$

where the stochastic discount factor for stage two firms is given by

$$Q^i(\tau'|\tau) = \beta\pi(\tau'|\tau)\frac{u'(C^i(\Lambda'))}{u'(C^i(\Lambda))}\frac{P^i(\Lambda)}{P^i(\Lambda')} \tag{12}$$

The aggregate state, S', evolves according to a law of motion that depends in today's aggregate state, S, and tariff shock, τ.

$$S' = F(S, \tau) \tag{13}$$

2.2 Equilibrium

To solve for the equilibrium of the dynamic model, this section describes the optimal sourcing decisions and pricing mechanism for the tradable goods of stage one and stage two. Then the market clearing conditions are defined.

2.2.1 Prices

Stage one prices. Stage two firms demand stage one goods for its production. Depending on the dynamic choice, m, only firms who have decided to import will have access to world market and source from lowest cost supplier in the economy. In this case, since stage one firms are perfectly competitive, the final price of the good for a stage two firm in country i, $p_1^i(z, \tau^t)$ will be equal to the minimum between the marginal cost of the firm in its country, $p_1^{ii}(z, \tau^t) = q_1^i(z, \tau^t)$, or the marginal cost times the tariff of the other country, j, $p_1^{ji}(z, \tau^t) =$

$\tau q_1^j(z,\tau^t)$

$$p_1^i(z,\tau^t) = \min\{p_1^{ii}(z,\tau^t), p_1^{ji}(z,\tau^t)\} = \{q_1^i(z,\tau^t), \tau q_1^j(z,\tau^t)\} \tag{14}$$

where $q_1^i(z,\tau^t)$ is the marginal cost of the stage one firm given by equation 7.

If stage two firm decides not to import, $m = 0$, it will have to source stage one good locally and the price will be given by the marginal cost of the local stage one firm, $q_1^i(z,\tau^t)$.

Stage two prices. Stage two goods are demanded by the representative household. The household in each country will source the variety z from the lowest cost supplier of the economy. There are two firms, one in each country, producing the same variety, z, and taking into account the demand they face from the final consumers. A stage two firm wants to obtain the monopoly price for selling its good z, but it faces the competition from the firm producing the same good in the other country. Given this duopoly structure, in equilibrium prices are obtained through the limit pricing result. This means that firms, in equilibrium, will not be able to charge more than the second-lowest cost of supplying the final household.

Positive levels of tariffs create different price levels for the stage two firm selling to both households. Thus creating the possibility for firms to impose different prices for each household.

Let $q_2^i(z,\tau^t)$ be the monopoly price of stage two firm in country i producing variety z,

$$q_2^i(z,\tau^t) = \frac{\epsilon}{\epsilon-1} \frac{w^i(\tau^t)^{1-\theta_2}\, p_1^i(z,\tau^t)^{\theta_2}}{(1-\theta_2)^{1-\theta_2}\, \theta_2^{\theta_2}\, A_2^i(z)} \tag{15}$$

which is equal to the marginal cost times the monopoly markup, $\frac{\epsilon}{\epsilon-1} > 1$.

Assuming firm in j supplies to household i, the price it sets, $p_2^{ji}(z,\tau^t)$, is the minimum between its monopoly price times the tariff, $\tau q_2^j(z,\tau^t)$, or the marginal cost of its competitor,

$((\epsilon-1)/\epsilon)q_2^i(z,\tau^t)$.

$$p_2^{ji}(z,\tau^t) = \min\{\tau q_2^j(z,\tau^t), ((\epsilon-1)/\epsilon)\, q_2^i(z,\tau^t)\} \tag{16}$$

If firm in country i supplies to its own household, then the price it sets, $p_2^{ii}(z,\tau^t)$, is the minimum between its monopoly price, $q_2^i(z,\tau^t)$, or the marginal cost times the tariff of its competitor, $((\epsilon-1)/(\epsilon)\tau q_2^j(z,\tau^t)$.

$$p_2^{ii}(z,\tau^t) = \min\{q_2^i(z,\tau^t), ((\epsilon-1)/\epsilon)\, \tau\, q_2^j(z,\tau^t)\} \tag{17}$$

Then the final price paid by household i, will be defined by its lowest cost supplier, given by

$$p_2^i(z,\tau^t) = \min\{p_2^{ii}(z,\tau^t), p_2^{ji}(z,\tau^t)\} \tag{18}$$

2.2.2 Market clearing equations

The market clearing for stage one and stage two goods for all z,τ^t

$$y_1^i(z,\tau^t) = \sum_j x_1^j(z,\tau^t)\mathbb{1}\Big(p_1^{ij}(z,\tau^t) \le p_1^{jj}(z,\tau^t)\Big) \tag{19}$$

$$y_2^i(z,\tau^t) = \sum_j x_2^j(z,\tau^t)\mathbb{1}\Big(p_2^{ij}(z,\tau^t) \le p_2^{jj}(z,\tau^t)\Big) \tag{20}$$

where the indicator function tells who the lowest cost supplier for market j is.

For the composite good, the demand from all stage one good firms in each country has to be equal to the production made by households for each history, τ^t.

$$N^i(\tau^t) = \int_0^1 N^i(z,\tau^t)dz \tag{21}$$

Labor market clearing for each country in each history, τ^t, takes into account the labor demanded in stage one, stage two, and the labor allocated towards fixed and sunk costs.

$$L_t^i = \int_0^1 l_1^i(z,\tau^t)dz + \int_0^1 l_2^i(z,\tau^t)dz + \int_0^1 m(z,\tau^t)\Big(f_1 + (1 - m_{\text{-}}(z,\tau^t))f_0\Big)dz \tag{22}$$

And last, the budget constraint of the household has to hold in equilibrium,

$$P^i(\tau^t)C^i(\tau^t) = w^i(\tau^t)L_t^i + \Pi^i(\tau^t) + T^i(\tau^t) \tag{23}$$

where $\Pi^i(\tau^t)$ are the aggregate profits of stage two firms net of fixed and sunk costs, and $T^i(\tau^t)$ are the aggregated tariffs obtained from all the imported goods from stage one and two.

Equilibrium. Given the parameters $\{\epsilon, \beta, \theta_1, \theta_2\}$, $\{A_1^i(z), A_2^i(z)\}_{\forall z}$, and $\{\tau^t\}_{\forall t}$ an equilibrium of the economy is a collection of prices $\{w^i(\tau^t), P^i(\tau^t), p_{1t}^i(z,\tau^t), p_2^i(z,\tau^t)\}_{\forall z,t}$, aggregate quantities $\{C^i(\tau^t), N^i(\tau^t)\}_{\forall t}$, production, sourcing, and input decisions $\{y_2^i(z,\tau^t),$ $y_1^i(z,\tau^t), x_2^i(z,\tau^t), x_1^i(z,\tau^t), l_1^i(z,\tau^t), l_2^i(z,\tau^t), N^i(z,\tau^t)\}_{\forall z,t}$, exogenous variables, $\{L_t^i\}_{\forall t}$, and a collection of the sequence of value functions, profit, import decisions, and aggregate states, $\{V^i(Z, m_{\text{-}};\tau, S), \Pi^i(Z, m;\tau, S), m(Z,\tau), S'(S,\tau)\}$ that satisfy the following conditions: (i) the household solve its maximization problem, (ii) stage one firms maximize their profits, (iii) stage two firms solve its static and dynamic problems, and (iv) the market clearing conditions hold.

2.3 Modeling uncertainty in trade policy

The objective of the paper is to isolate the effect uncertainty in future tariffs has under the structure of a multistage dynamic model. To do so, the analysis must separate the effect of the change in levels of the tariffs, first moment shocks, and the second moment shock, the uncertainty regarding the tariff change.

The paper partially follows Steinberg B. (2019) methodology. He compares the stochastic equilibrium path with a perfect foresight equilibria evaluated among all the possible tariff paths. Instead this papers proposes to evaluate the perfect foresight equilibrium along the expected value of the tariffs used in the stochastic equilibrium. With this addition the uncertainty component is further isolated from the first moment shock. In the next section, a simple exercise is proposed and the numerical results analyzed.

2.4 Discussion

Prior to discussing how to translate this model into a quantitative framework for analysis, following are comments on some aspects of the model.

Multistage and roundabout production. First, the model features sequential multistage and roundabout production. Roundabout production is present through the composite good, which is produced aggregating over all stage two varieties, and used as input in stage one production. This introduces a loop in the production process, which amplifies the ratio of gross output to value added. That is, gross output will exceed value added both because of multistage production implies that inputs are produced and used up in the production process, but also because production in each sector used its own output as inputs. This feature creates input linkages across sectors in the model, which are important to calibrate the model to match the data[7].

Second, the multistage aspect of the model is essential to understanding the behavior of the response of trade flows and aggregate variables to trade costs. Stages of production are more often located in the same country when trade costs are high. As trade costs fall, it is increasingly attractive to exploit cost differences and break up production stages across

[7] This result of roundabout production in a multistage environment has been explored by Yi (2003), Yi (2010), Johnson C. and Moxnes (2019), Antrás and De Gortari (2017), among many others.

countries. The ability to substitute over the location of individual stages, rather than the location of the entire final good, amplifies the sensitivity of trade flows to trade costs.

Limit pricing. Each variety $z \in [0, 1]$ has a stage one and a stage two good. For each stage one good z, each country has a perfectly competitive sector. For stage two good z, there is one firm in each country competing for the final consumption market. Because we have one firm in each country producing the same stage two good, z, firms act as a duopoly. Thus, the prices for stage two good z follow the limit pricing result discussed in the section above. If a firm supplies a given household, it gan either price its monopoly price, or the marginal cost of its competitor adjusted by tariffs. Thus, tariffs not only move the location of the production of goods, but also affect the pricing decisions of the stage two firms. Limit pricing affects the general equilibrium in two different ways.

First, stage two prices will increase as tariffs increase. But this increase will be nonlinear, due to the decrease in the level of competition the stage two firm faces against its competitor in the other country. Second, the firm's dynamic choice, whether to import stage one goods or not, depends on the profits made by the firms. A change in tariffs will change profits directly, and indirectly through the change in prices.

Dynamic choice. Stage two firms face the dynamic choice whether to import stage one goods and pay a one time only sunk cost. To continue with their importer status, firms need to pay each period a fixed cost.

The introduction of the sunk cost allows for future tariffs to affect firm decisions' today. If firms have complete information regarding the future tariff path, they would know exactly when to either make the investment and import, or when to stop paying the per period fixed cost and lose their importer status.

In the case with trade policy uncertainty, when firms don't know for certain the tariff path, sunk costs can generate a higher option value of waiting[8]. Given a low tariff shock, firms

[8]This channel has been studied by Dixit (1989), Roberts J. and Tybout R. (1997), Handley (2014), among

will delay the decision to start importing due to a possible future tariff increase. Similarly, given a high tariff shock, firms will delay the decision to stop importing due to a possible future tariff decrease.

Aggregate state. An important part of solving the model is solving for the aggregate state of the economy, S. The aggregate state can be represented as a vector of cutoff levels that indicate the marginal stage two firm willing to import the stage one good. The framework presented includes multistage production in a fixed unit measure of goods and limit pricing that make for more than one marginal firm willing to import[9].

Multistage production in a fixed unit measure of goods creates more than one cutoff because one variety z has a vector $Z = \{A_2^1(z), A_2^2(z), A_1^1(z), A_1^2(z)\}$ of associated productivities. For stage two firms, its overall firm productivity not only depends on its own production productivity $A_2(z)$, but on the productivity of the stage one firm it decides to source from, $A_1(z)$. Whether a stage two firms produces or not, depends on the distance of the firm's overall productivity from its competitor's. A stage two firm could have a high level of productivity, but if its lower than its competitor's it might never produce.

The limit pricing result also induces additional cutoff levels for stage two firms. Stage two firms are constrained not to charge more than the second-lowest cost of supplying the market. If a firm sources to a given household, then it can either price it's monopoly price (maybe adjusted times the tariff depending on the location of the household and firm), or the marginal cost of its direct competitor. Then the profits the firm obtains from a given household, used to pay the fixed and sunk cost of importing, could be a function of the firm's own overall productivity, or of its competitor's, depending on the firm's relative productivity.

Both of this features of the model create different cases or areas within the unit measure of

others.

[9] Papers with dynamic one stage models with no head to head competition have only one cutoff productivity level that is able to summarize the aggregate state of the economy CITE.

goods, each with its own cutoff level, or marginal firm, that defines which stage two firms import stage one goods from the other country. The next section will describe how the cutoffs are obtained, and shed more light on how the different cases arise from the problem.

The aggregate state of the economy tomorrow, S' [10], will depend on the productivity vectors associated to each variety $Z = \{A_1^1(z), A_1^2(z), A_2^1(z), A_2^2(z)\}_{\forall z}$, aggregate prices today, $w^i(\Lambda)$ and $P^i(\Lambda)$, and today's tariff shock, τ.

The larger the vector of cutoff levels, S, the more computationally challenging solving the problem is. Ideally, this paper would solve the problem as presented, but first additional assumptions will be imposed to simplify the aggregate state, S. Then preliminary results will be presented for this simplified environment. Note that to be able to take full advantage of the structure of the model, the paper will have the solve for the model without the simplifying assumptions.

3 Solving the model

The next section describes the additional assumptions for the simplified environment, and outlines the solution procedure used to obtain numerical results.

3.1 Simplifying Assumptions

To simplify the aggregate state, S, two assumptions will be made. First, assume there are no tariffs for importing stage two goods. Tariffs will only apply to stage one good imports. This allows for perfect specialization in both stages of production. As $\tau = 0$ for stage two imports, the price the households pay for this goods is equal in both countries. Then there will be a unique stage two firm supplying to both households. This in turn implies that the

[10] S' represents the firms that imported today and survived the δ shock tomorrow

unique stage two firm will supply from a unique stage one firm.

Additionally, assume there is no limit pricing result for stage two goods. Assume that the lowest cost supplier of stage two goods supplies to both markets and is able to price its monopoly price, regardless of its competitor's productivity.

$$p_2^i(z, \tau^t) = \min\{q_2^i(z, \tau^t), q_2^j(z, \tau^t)\} \tag{24}$$

where $q_2^i(z, \tau^t)$ is the monopoly price given by equation 15.

With this assumptions, stage two prices are the same for both households, which implies that the price index for the final and composite good is equalized among countries, $P^i(\tau^t) = P^j(\tau^t)$. Also, stage two prices will be a function of the producing firm's productivity, and never of its competitor's. The aggregate state of the economy, S, in this simplified environment is composed of four cutoff levels. Four different cases, each with its own stage two marginal firm that is willing to import. The next section will describe the methodology used to pin down the cutoff levels.

Note that with this assumptions, some important frictions are lost. Eliminating the tariffs for stage two goods reduces the model's ability to explain the response of trade flows to changes in trade costs. This advantage of using multistage production vs a one stage model is partially lost. Additionally, assuming away the limit pricing result eliminates an interesting a mechanism where tariffs change the prices and profits of the firms.

3.2 Solving for the aggregate state of the economy

The aggregate state of the economy, S, is given by the distribution of the import status the stage two firms enter the period with. This is given by the firm's decision to import the period before and the probability $(1-\delta)$ that the stage two firm maintains it's import status. Given the simplifying assumptions, the aggregate state, S, is reduced to a vector with four cutoff levels.

The cutoff levels are found by the marginal firm willing to import, i.e. the stage two firm that is indifferent between importing stage one good or sourcing it locally. This firm can be found by rewriting the stage two firm's dynamic choice problem:

$$V^i(Z, m_-; \Lambda) = \max\{V^{i1}(Z, m_-; \Lambda), V^{i0}(Z, m_-; \Lambda)\} \tag{25}$$

where $\Lambda = (\tau, S)$, $V^{i1}(Z, m_-; \Lambda)$ represents the value when the firm decides to import, $m = 1$, and $V^{i0}(Z, m_-; \Lambda)$ the value when the firm decides not to import, $m = 0$.

$$\begin{aligned} V^{i1}(Z, m_-; \Lambda) &= \Pi^i(Z, 1; \Lambda) - w^i(\Lambda)\Big((1 - m_-)f_0 + f_1\Big) \\ &\qquad + \delta \sum_{\tau'} Q^i(\tau'|\tau)V^i(Z, 0; \Lambda') + (1 - \delta) \sum_{\tau'} Q^i(\tau'|\tau)V^i(Z, 1; \Lambda') \\ V^{i0}(Z, m_-; \Lambda) &= \Pi^i(Z, 0; \Lambda) + \sum_{\tau'} Q^i(\tau'|\tau)V^i(Z, 0; \Lambda') \end{aligned} \tag{26}$$

For each value of $m_- = \{0, 1\}$, the cutoff level is pinned down by the marginal stage two firm producing variety z, whose value functions are equalized $V^{i1}(Z, m_-; \Lambda) = V^{i0}(Z, m_-; \Lambda)$. To find this marginal firms, the profits of the firms who would like to import have to be analyzed.

A stage two firm in country i would like to import if the world price for stage one good is lower than the price of the stage one firm in its own country, $p_1^i(Z; \Lambda) < q_1^i(Z; \Lambda)$. If a stage two firm z decides to import today, then its profits are given by:

$$\Pi^i(Z, 1; \Lambda) = \Gamma^i \left(\frac{w^j(\Lambda)^{(1-\theta_1)\theta_2} \ \tau^{\theta_2}}{A_2^i(z) A_1^j(z)^{\theta_2}} \right)^{1-\epsilon} \tag{27}$$

where Γ^i is a factor that affects all stage two firms in country i similarly.

$$\Gamma^i = \frac{1}{\epsilon}\left(\frac{\epsilon\, w^i(\Lambda)^{1-\theta_2} P^i(\Lambda)^{\theta_1\theta_2}}{\Theta_2\Theta_1^{\theta_2}(\epsilon-1)}\right)^{1-\epsilon}\left(\sum_{j=1}^{2} P^j(\Lambda)^{\epsilon}(C^j(\Lambda)+N^j(\Lambda))\right) \tag{28}$$

where $\Theta_n = (1-\theta_n)^{1-\theta_n}\,\theta_n^{\theta_n}$, for $n \in \{1,2\}$.

If a stage two firm z decides to not import today, then the profit it obtains $\Pi^i(Z,0;\tau,S)$ depends on whether the firm can still supply to the final households or not.

$$\Pi^i(Z,0;\Lambda) = \begin{cases} \Gamma^i\left(\dfrac{w^i(\Lambda)^{(1-\theta_1)\theta_2}}{A_2^i(z)A_1^i(z)^{\theta_2}}\right)^{1-\epsilon} & \text{firm still wins stage two market} \\ 0 & \text{firm loses stage two market} \end{cases} \tag{29}$$

Thus, for a given state, $\Lambda = (\tau, S)$, and import status, m_-, there are two different cutoff levels per country. Cutoff levels are defined by

$$V^{i1}(Z_c^i, m_-;\Lambda) = V^{i0}(Z_c^i, m_-;\Lambda) \tag{30}$$

which is equivalent to

$$\Pi^i(Z_c^i,1;\Lambda) - \Pi^i(Z_c^i,0;\Lambda) + (1-\delta)\sum_{\tau'} Q^i(\tau'|\tau)\Big(V^i(Z_c^i,1;\Lambda') - V^i(Z_c^i,0;\Lambda')\Big)$$
$$= w^i(\Lambda)\,\Big((1-m_-)f_0 + f_1\Big)$$

Among the firms who want to import, $\forall z$ s.t. $p_1^i(Z;\Lambda) < q_1^i(Z;\Lambda)$, the two marginal firms in each country are:

1. Wants to import and wins stage two market without importing, the marginal firm, Z_1^i, is given by:

$$V^{i1}(Z_1^i, m_-;\Lambda) = V^{i0}(Z_1^i, m_-;\Lambda) \tag{31}$$

which is equivalent to

$$\Gamma^i\left(\left(\frac{w^j(\Lambda)^{(1-\theta_1)\theta_2}\ \tau^{\theta_2}}{A_2^i(z)A_1^j(z)^{\theta_2}}\right)^{1-\epsilon}-\left(\frac{w^i(\Lambda)^{(1-\theta_1)\theta_2}}{A_2^i(z)A_1^i(z)^{\theta_2}}\right)^{1-\epsilon}\right)$$
$$+(1-\delta)\sum_{\tau'}Q^i(\tau'|\tau)\Big(V^i(Z_1^i,1;\Lambda')-V^i(Z_1^i,0;\Lambda')\Big)=w^i(\Lambda)\Big((1-m_-)f_0+f_1\Big)$$

and the marginal firm is given by

$$Z_1^i=\left(\frac{w^j(\Lambda)^{(1-\theta_1)\theta_2}\ \tau^{\theta_2}}{A_2^i(z)A_1^j(z)^{\theta_2}}\right)^{1-\epsilon}-\left(\frac{w^i(\Lambda)^{(1-\theta_1)\theta_2}}{A_2^i(z)A_1^i(z)^{\theta_2}}\right)^{1-\epsilon} \tag{32}$$

From the stage two firms that want to import, all firms $z \geq Z_1^i$ will do so.

2. Wants to import and lose stage two market without importing, the marginal firm, Z_2^i is given by:

$$V^{i1}(Z_2^i,m_-;\tau,S)=V^{i0}(Z_2^i,m_-;\tau,S) \tag{33}$$

which is equivalent to

$$\Gamma^i\left(\frac{w^j(\Lambda)^{(1-\theta_1)\theta_2}\ \tau^{\theta_2}}{A_2^i(z)A_1^j(z)^{\theta_2}}\right)^{1-\epsilon}+(1-\delta)\sum_{\tau'}Q^i(\tau'|\tau)\Big(V^i(Z_1^i,1;\Lambda')-V^i(Z_1^i,0;\Lambda')\Big)$$
$$=w^i(\Lambda)\Big((1-m_-)f_0+f_1\Big)$$

and the marginal firm is given by

$$Z_2^i=\frac{1}{A_2^i(z)A_1^j(z)^{\theta_2}} \tag{34}$$

From the stage two firms that want to import, all firms $z \geq Z_2^i$ will do so.

For the simplified environment, the aggregate state is given by, $S=\{Z_1^1,Z_1^2,Z_2^1,Z_2^2\}$.

3.3 Solution Method

This section describes a method for discretizing the unit measure of goods, $z \in [0, 1]$, and then outlines the procedure used to solve the model.

3.3.1 Approximating indicator functions

To numerically solve the model, the unit measure of goods needs to be discretized. To do so, it's assumed there is a large, finite number of stage one and two goods, indexed by $r = \{1, 2, ..., R\}$. The standard approach is to approximate the unit measure with a rather large value for R[11]. This is computationally expensive, so instead I follow Johnson C. and Moxnes (2019), and introduce an approximation of the indicator function that allows for a smaller value for R[12].

The indicator functions in the market clearing for goods in stage one and stage two are approximated using the logit-smoothed accept-reject (AR) simulator, developed by McFadden (1989). The logit-smoothed AR simulator approximates the indicator function with a continuous logit function,

$$\mathbb{1}\Big(p_n^{ij}(r, \tau^t) \leq p_n^{jj}(r, \tau^t)\Big) \approx \frac{e^{-p_n^{ij}(r,\tau^t)/\lambda}}{\sum_k e^{-p_n^{kj}(r,\tau^t)/\lambda}} \tag{35}$$

where $\lambda > 0$ is a smoothing parameter that determines the accuracy of the approximation of the indication function[13].

[11] Yi (2010) approximates the unit measure with a R equal to 1.5 million.

[12] Johnson C. and Moxnes (2019) introduce this assumption and solve their multistage model with an R = 20,000.

[13] As $\lambda \to 0$ the logit function converges to the indicator function. The choice of λ is guided by a trade-off between accuracy and computational speed, and there is little guidance on the appropriate level of λ in general. Johnson C. and Moxnes (2019) find by trial and error, that $\lambda = 0.1$ yields a very good approximation to the exact equilibrium.

3.3.2 Solution procedure

Uncertainty in this environment is modeled the following way. The economy starts in a steady state where tariffs are constant. Then there is a period where the economy experiences stochastic tariff shocks, but eventually encounters an absorbing final state. Finally, the economy converges to a final steady state. A brief overview of the procedure used to solve the model is as follows.

Given parameters $\{\epsilon, \beta, \theta_1, \theta_2\}$, data $\{L_t^i\}_{\forall t}$, productivity draws, $\{A_n^i(z)\}_{\forall R,n,i}$, the first step is to solve for the initial and final steady states. Then a guess for the aggregate state S_t along all the transition path, T, is proposed. The transition path includes the period with stochastic tariff shocks, and the transition period when the economy converges to the final steady state. Given the guess for the aggregate state along the transition path, aggregate variables are computed. Using backward induction, the value and policy functions for all stage two firms are calculated. The guess for S_t is compared with the policy functions. If they don't coincide, then the guess for S_t is updated. Finally, make sure in period T variables coincide with the final steady state. If not, increase T. A more detailed algorithm is described in Appendix A.

4 Results

The following section describes the exercise that will be solved numerically. It includes the parameter values used for the solution, and then reports the results obtained.

4.1 Modeling Uncertainty

In the simplified environment described, the following exercise is numerically solved using the procedure described in the last section. As described before, the objective is to isolate the effect uncertainty in future tariffs has in the aggregate variables. The stochastic equilibrium exercise will be compared to a perfect foresight equilibrium evaluated at the expected value of the tariffs.

The exercise has one period of uncertainty. In $t = 2$ an unexpected announcement is made, where tariffs could either increase or decrease with equal probability in the next period. And then tariffs remain constant from $t = 3$ after. In $t = 3$, with $\pi = 0.5$ tariffs increase to $\tau_H = 10\%$, and with $1 - \pi = 0.5$ tariffs decrease to $\tau_L = 0\%$.

$$\tau^t = \begin{cases} \tau_M = 5\% & t = 1 \\ \tau_M = 5\% & t = 2 \text{ an announcement is made} \\ \tau_H = 10\%,\ \tau_L = 0\% & t \geq 3 \end{cases} \tag{36}$$

Perfect foresight equilibria are computed, one where the tariff increases to τ_H, other where it decreases to τ_L, and lastly where they remain constant in τ_M. The stochastic equilibrium should be compared to the path where tariffs remain in τ_M.

4.2 Assigned parameter values

The following table presents parameter and other variable values that have been taken out from the literature. Ideally, some of this parameters will be calibrated to fit the data from the recent trade policy uncertainty increase presented before.

The productivities for stage one and stage two firms are independent draws form a Frechet distribution,

$$G(A_n^i(z)) = e^{-T_n^i A_n^i(z)^{-\eta}} \qquad n \in \{1, 2\} \tag{37}$$

where a higher T_n^i implies a higher efficiency draw is more likely and η governs the level of heterogeneity of the draws.

Table 1: Parameter Values

Parameters	Value	Comment
$A_1^i(r), A_2^i(r)$		iid draws from Frechet: $F(A_n^i(r)) = e^{-T_n^i A_n^i(r)^{-\eta}}$
T_1^1, T_2^1	$1, 1.2$	country one comparative advantage in stage two
T_1^2, T_2^2	$1.2, 1$	country two comparative advantage in stage one
η	6	degree of heterogeneity of the draws
ϵ	4	elasticity of substitution for stage 2 goods
θ_1	0.4	share of intermediate goods in stage 1
θ_2	0.6	share of stage 1 goods in stage 2 production
β	0.98	discount factor
L_1, L_2	1	normalized labor supply
w^1	1	price normalization
f_1	0.0002	represents 1% of sales of the average firm
f_0	0.015	represents 60% of sales of the average firm
R	100	number of firms (Jhonson and Moxnes (2019) used 20,000)
λ	0.005	smoothing parameter, lower than Jhonson and Moxnes (2019)

4.3 Results

Results are summarized in figures 4 through 7, and analyzed below.

In the simplified environment with the exercise presented above, the stage two firms in the stochastic equilibrium decide to wait until the uncertainty is resolved to make investment decisions. They wait until $t = 3$ to take action. Them waiting has a cost for the economy. In the perfect foresight equilibrium, firms start modifying their dynamic choice since period $t = 2$, which enables the households to smooth their consumption.

A richer exercise for uncertainty, with multiple periods of uncertainty and tariff shocks, under the full structure of the model, is expected to show other mechanisms trade policy uncertainty is costly for the economy. Stage two firms could chose to make import investment, and never import. Or firms who shouldn't be importing could continue to pay fixed costs for multiple periods. This type of scenarios would provide additional costs for the economy, and allow for a better analysis of the effect trade policy uncertainty has in the economy.

Tariffs decrease. In the perfect foresight equilibrium, stage two firms anticipate the change in tariffs and start changing their dynamic choice from period $t = 2$. As observed in figures 4 and 5, when tariffs decrease, more firms chose to import. At δ rate firms lose their status as importers, so every period there are payments of the sunk cost, in addition to the new firms starting to import due to the tariff change. Payment of this sunk and fixed costs will increase the demand for labor and push wages up. Knowing this, some firms will chose to make the sunk cost investment in $t = 2$ and obtain their status as importers when tariffs are still high, since wages are expected to increase in $t = 3$.

This anticipation effect is present in country one and country two, but since country two has comparative advantage in the production of stage two, more firms will choose import in country two. Note there is also a transition period after the uncertainty is resolved. The payment of the sunk costs increases the wages and lowers the consumption levels, so not all firms can start importing the same period. The firms that have the most incentives to import will do so first, and the rank continues until the economy converges to the final steady state.

In the stochastic equilibrium, firms decide to wait until the uncertainty is resolved next period to take action. Due to trade policy uncertainty, stage two firms rather wait and pay the sunk cost after, even though wages are higher making the investment more expensive. Similarly to the perfect foresight case, there is a transition period after the uncertainty is resolved, where the economy converges to the steady state.

As tariffs increase, two forces affect consumption. On one hand an increase in tariffs makes firms less productive, which decreases the aggregate consumption level. On the other hand, the increase in tariffs decreases the number of firms paying the sunk and fixed cost every period, which increases the level of consumption. Figures 6 and 7 depict the trajectories of the consumption levels for country one and two. Plotted is the change in consumption relative to the case where tariffs remain in τ_M.

Country one has comparative advantage in stage two, and thus has a bigger percentage of importer firms. As tariff increase, consumption increases because the effect of the payments of the costs associated with importing is greater than the productivity loss of the firms. The opposite happens to country two. Since not many firms are importing, that effect of the payment of sunk and fixed costs is smaller than the productivity loss the firms face, which lowers the aggregate consumption of the country.

In the perfect foresight path, stage two firms know the change in tariffs and can smooth consumption in the transition path. In the stochastic equilibrium path, since firms opt to wait until uncertainty is resolved to take action, the consumption path observes bigger changes.

Tariffs increase. When tariffs increase, the firms with the least incentives to import will decide to stop. In the perfect foresight equilibrium, some firms stop importing in period $t = 2$, even though there is no change in tariffs. This firms experienced the δ shock in $t = 2$, which removed its status as importers, and decided not to pay the sunk cost again. They do so because tariffs increase next period, and they don't find it optimal to import anymore. This phenomenon is present in both countries, as depicted in figure 4 and 5. Note that when tariffs increase, there is no transition period to converge to the final steady state, since the decision to stop importing has no associated fixed or sunk costs.

When there is uncertainty in future tariffs, firms decide to wait. Even though firms experience the δ shock, the possibility of the tariff decrease pushes firms to pay the sunk cost in period $t = 2$, even though they will lose their status next period when the tariff increases.

Regarding consumption, the two opposite forces described above are present. Trajectories for teh change in consumption relative to the consumption level when tariffs don't change can be observed in figures 6 and 7. When tariffs increase, less firms want to import. Decrease in payments of fixed and sunk costs pushes consumption up. But the decrease in productivity associated to the increase in tariff's effect is greater, and this causes the overall decrease in consumption present in both countries. Along the perfect foresight equilibrium consumption is smoothed out, relative to the uncertainty equilibrium.

Exercise limitations. Note that the current exercise only has one period uncertainty about the tariff tomorrow, and there is no tariff shock and uncertainty in the same period. An economy with multiple periods of uncertainty with tariff shocks will have more than anticipation affects.

A tariff shock today paired with tariff uncertainty tomorrow, will allow for real costs in firms decisions. If tariffs decrease today, but tariffs are uncertain tomorrow, a firm may not chose to import today, even though it would do so if there was perfect foresight. Given a tariff increase today with tariff uncertainty tomorrow, a firm may continue to pay for the fixed cost to keep on with its importer status, even though it wouldn't do so if the uncertainty wasn't present.

Decision made under uncertainty will be costly for firms and households. Not only due to the inability of firms to smooth out consumption, but because firms could make decision today that turn out to be incorrect once the future tariff shocks are realized.

5 Conclusion

New trade deals, trade wars, and renegotiations of trade agreements have brought attention to new studies regarding trade policy uncertainty. This emerging literature has analyzed trade policy uncertainty under the lens of a one stage production model. In contrast, this paper analyzes uncertainty in future tariffs in a multistage production environment. Multistage production is able to generate a magnified response of trade to tariff changes. It allows for a better mechanism to analyze the effect uncertainty has in industries whose final good is produced in multiple stages that are located across different countries.

In a stochastic environment, where tariffs are uncertain, and with sunk costs firms must pay in order to import some of its inputs, there is an option value of waiting. Firms will delay their decision to start importing, due to a possible increase in tariffs. Or firms will delay exit from the import market, expecting tariffs to decrease in the future. Waiting is costly. In the simplified environment presented, it prevent households from smoothing consumption and delays investment.

Next steps will be to use the full structure of the model under a rich uncertainty environment. Multi-period uncertainty with constant tariff shocks will show additional mechanisms uncertainty affects the economy. Mechanisms that could have greater costs. Stage two firms may chose to become importers, pay the associated sunk cost, and never be able to import. Or, in equilibrium, the economy could sustain firms paying the fixed costs, and not importing stage one goods. I expect this model will provide a better scope to analyze trade policy uncertainty.

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A Solution Procedure

The following detailed overview of the procedure includes solving for the steady states and the transition path, given the specific stochastic path for tariffs, $\{\tau^t\}_{\forall t}$, with its associated probabilities $\pi(\tau'|\tau)$.

First step. Given parameters $\{\epsilon, \beta, \theta_1, \theta_2\}$, data $\{L^i_t\}_{\forall t}$, productivity draws, $\{A^i_n(z)\}_{\forall R,n,i}$, the first step is to solve for the initial and final steady states. Each steady states has its associated constant tariff. Computing steady states requires its own procedure.

1. Guess for the cutoff levels of S_g, consumption levels, C^i_g, aggregate composite good, N^i_g, price index, P^i_g, and wages, w^i_g.

2. Using the price index in equation 2, iterate until you find the price index, P^i.

3. Using the market clearing in equation 21, iterate until you find the fixed point for the composite good, N^i.

4. With the labor market clearing update and iterate until we obtain the fixed point for the wage in country two, w^2. Wage in country one is used as the price normalization, $w^1 = 1$[14]. Each iteration updates the fixed points for P^i and N^i with the new value for the wages.

5. Iterate the budget constraint of the household to solve for the fixed point value of final consumption, C^i. In each iteration the fixed point for P^i, N^i, w^i has to be updates as

[14]Labor market clearing in country one is dropped appealing to Walras' Law.

well.

6. Once we have the fixed point values for P^i, N^i, w^i, and C^i for the guess S_g, we have to compute the value function and policy functions of all stage two firms from equation 11.

7. Compare the policy functions with the guess, S_g. If they are the same, stop. Otherwise update the guess with the result from the policy functions and start again from step 1.

Second step. Propose a guess for all the cutoff levels in the aggregate state along the entire transition path, with length T, $\{S_T\}_{t=1}^{T}$. The transition path includes the period of stochastic tariff shocks, and the time it takes to converge to the final steady state once tariffs are constant. The guess for the cutoff levels, S_t, should be an educated guess that goes with the specific tariff shocks that will happen in the transition path. The guess is composed by all the firms who want to start or stop importing, depending on the tariff shocks and final steady state. First, firms are ranked according to their willingness to import. Then, if the tariff decreases, using this rank every period a small set of firms are included to the set of importer firms. Or, if the tariff increases, using the inverse rank, a small set of firms each period is removed from the set of importer firms.

Third step. Once the educated guess for the aggregate state S_t along the transition path is made, the implied aggregate variables $\{Y_t, P_t, M_t, w_t\}_{t=1}^{T}$ are computed for entire transition path. Aggregate variables are calculated using the steps 1 through 5 described in the first step.

Fourth step. Using backward induction, the value and policy functions of all the stage two firms along the transition path are computed.

Fifth step. Compare the results from the policy function to the guess for the aggregate state, S_t. Add or remove firms each period depending on the rank computed in step two and the policy functions obtained, and update the guess for the cutoff levels.

Sixth step. Check whether the period T variables coincide with the final steady state. If not, increase T and redo the procedure.

B Figures

Figure 1: Share of importers in country 1

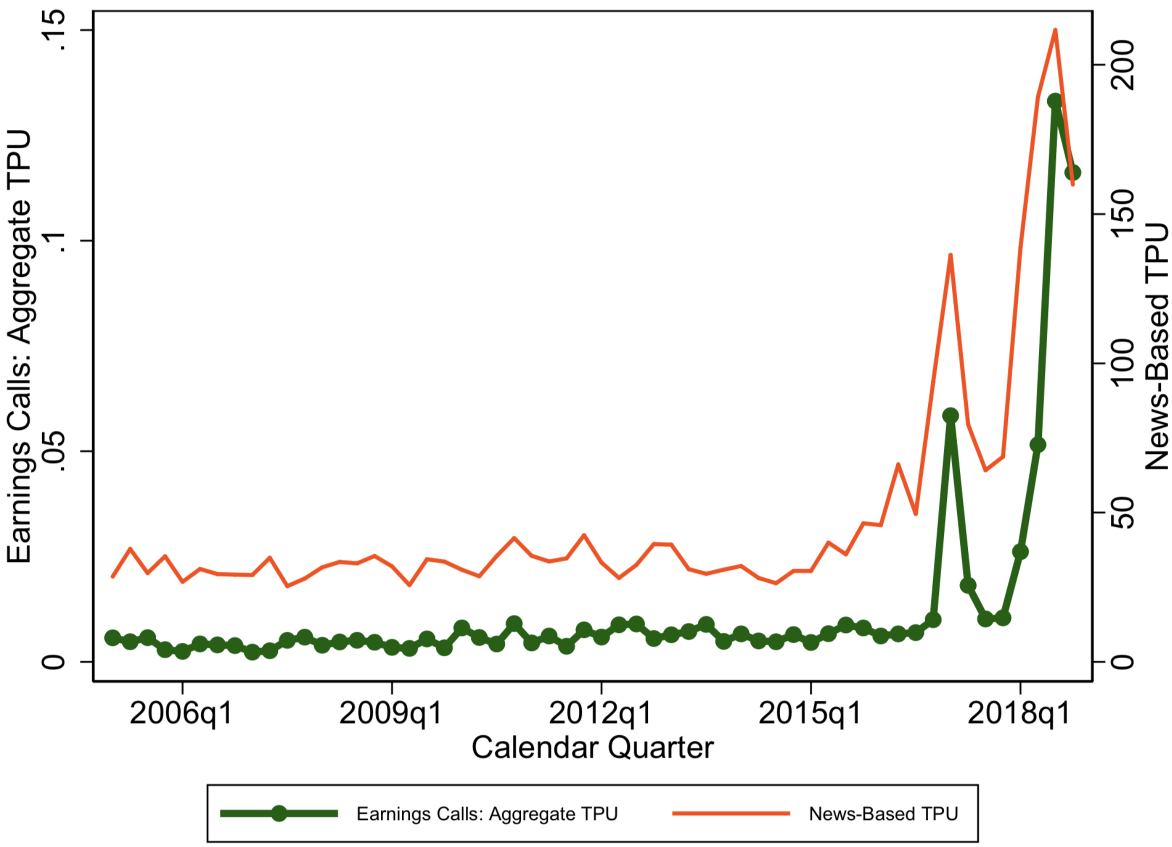

Figure 2: Share of importers in country 1

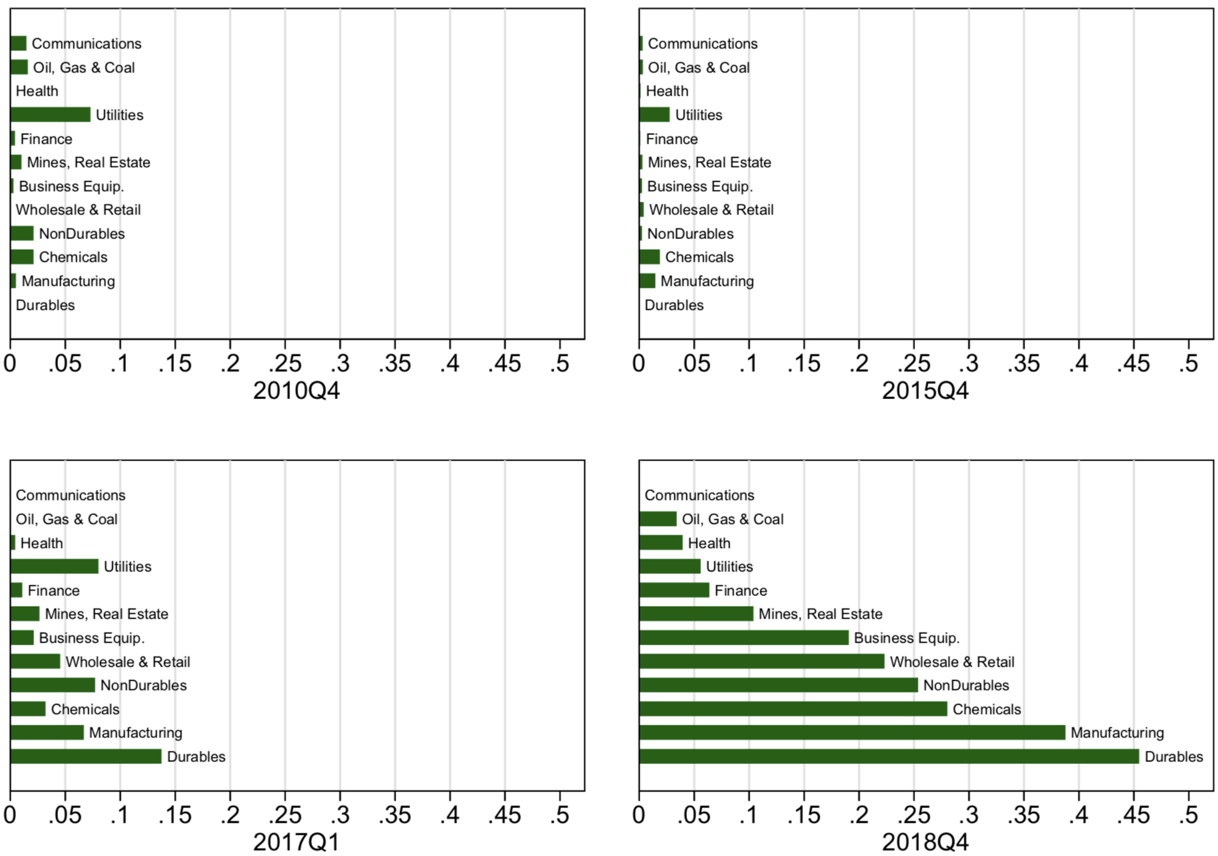

Figure 3: Share of importers in country 1

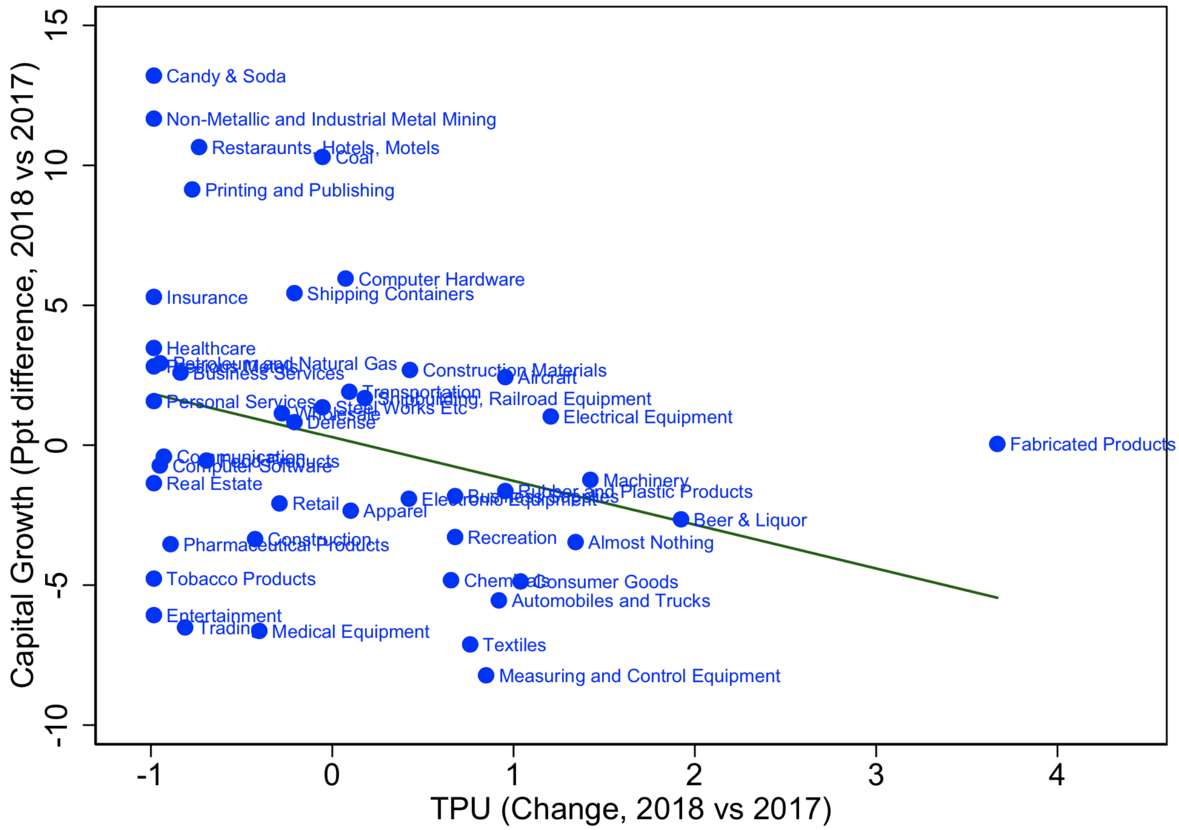

Figure 4: Share of importers in country 1

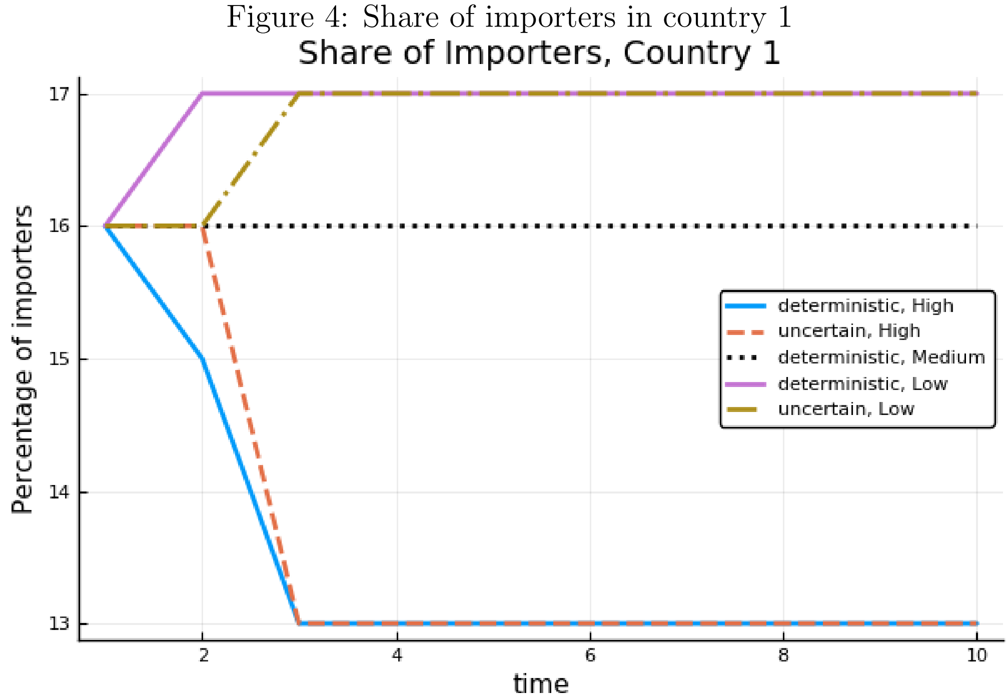

Figure 5: Share of importers in country 2

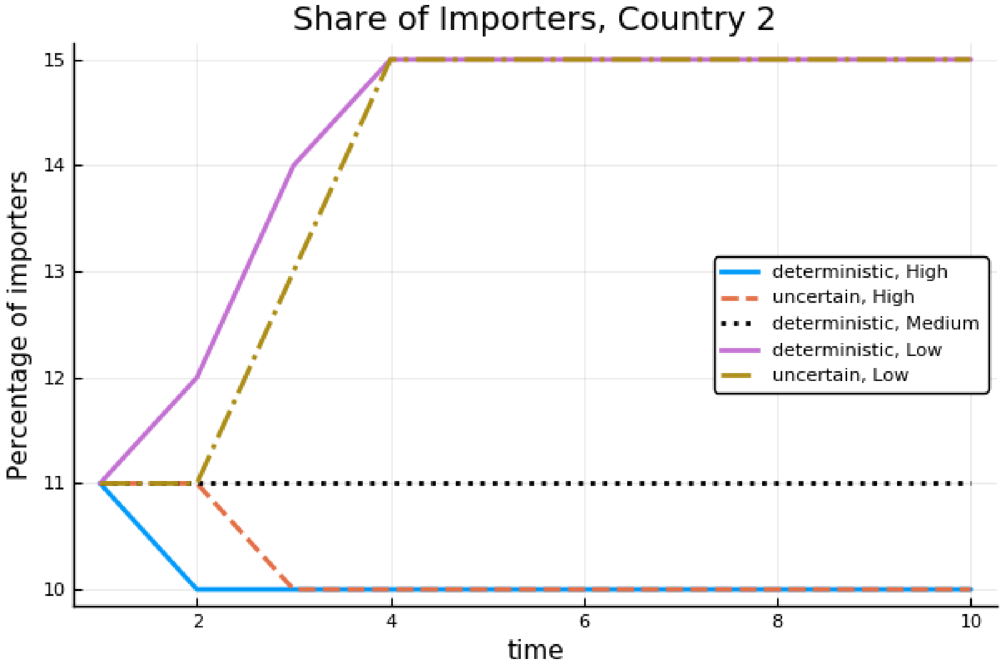

Figure 6: Change in consumption levels in country 1

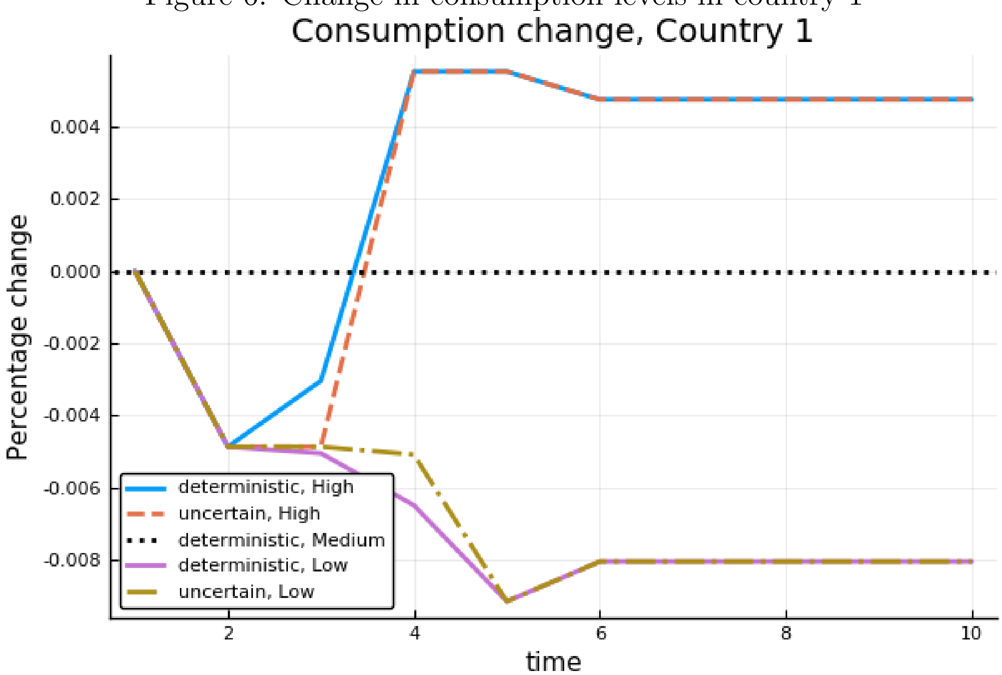

Figure 7: Change in consumption levels in country 2